

**SECOND SEMESTER 2023-2024**

**09-01-2024**

# Course Handout - Part II

In addition to Part-I (General Handout for all courses appended to the time table), this portion gives further specific details regarding the course.

*Course No.* : MATH F244

## Course Title : Measure & Integration

## Instructor-in-Charge : SHARAN GOPAL

*Instructors* : SHARAN GOPAL, Debarati Mondal and Shayon Bhadra

**1. Scope and Objective of the Course:** The objective of this course is to give a comprehensive and sound introduction to Lebesgue measure theory and integration. The concepts of several notions of convergence and convergence theorems are also covered in this course. The classical theory of Riemann integration has some obvious draw backs: Firstly, the class of Riemann integrable functions is relatively small and secondly the limiting operations often lead to insurmountable difficulties. In this course, the students will be taught, how to resolve these problems in the case of Lebesgue measure theory.

**2. Textbook:**

H. L. Royden, P. M. Fitzpatrick, Real Analysis, 4th Edition, Pearson Education India, 2015.

**3. Reference books**

1. G. de Barra, *Measure Theory and integration*, New Age International Ltd, Delhi, 2003.
2. P.K. Jain, V.P. Gupta, P. Jain, *Lebesgue Measure And Integration*, New Age International Ltd,

Delhi, 2nd ed., 2011.

1. Inder Kumar Rana : *Introduction to Measure & Integration*, Narosa, Delhi 1997.

**4. Course Plan:**

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| --- | --- | --- | --- |
| **Lecture No.** | **Learning objectives** | **Topics to be covered** | **Chapter in the Text Book** |
| 1-2 | To make the students understand that it is impossible to define a measure for all subsets of real numbers. | Length of an interval, Outer measure. | **2.1-2.2** |
| 3 –5 | To introduce the concepts of measurable sets and study the properties of Measurable sets | Lebesgue measurable sets and its properties, Borel sets and their measurability, Approximation of measurable sets. | **2.3 – 2.4** |
| 6-7 | To introduce Lebesgue measure, study its properties and introduce the idea of “almost everywhere”. | Lebesgue measure and its properties, The Borel-Cantelli Lemma | **2.5** |
| 8 | To prove the existence of  non-measurable sets. | Non-measurable sets | **2.6** |
| 9-11 | To define the Cantor set and show the existence of a non Borel subset of the Cantor set | The Cantor Set and the Cantor-Lebesgue Function | **2.7** |
| 12-14 | To study measurable functions | Definition and properties of measurable functions, Operations on measurable functions. | **3.1** |
| 15-17 | To study the measurability of limits of sequence of functions under various notions of convergence and then different approximations of measurable functions. | Pointwise limits and simple approximation. Littlewood's three principles, Egoroff's theorem, and Lusin's theorem | **3.2-3.3** |
| Proofs for Egoroff's theorem, and Lusin's theorem – Self study |
| 18-23 | To study the Lebesgue  Integral in various forms and its properties. | Review of Riemann integral, Lebesgue integral of a bounded function and its properties, Integrals of a non-negative measurable functions, General Lebesgue integrals and its properties | **4.1-4.5** |
| 24-25 | To give a characterization of Riemann and Lebesgue integral functions | Characterizations of Riemann and Lebesgue Integrability | **5.3** |
| 26-28 | To study the concept of a new notion of integrability, namely the uniform integrability | Uniform integrability, The Vitali convergence theorem, A general Vitali convergence theorem | **4.6, 5.1** |
| 29-30 | To study a new notion of convergence of sequence of functions | Convergence in Measure | **5.2** |
| 31-37 | To define differentiability and study the relationship between Integration and Differentiation | Continuity of Monotone Functions, Differentiability of Monotone Functions, Functions of Bounded Variation, Absolutely Continuous Functions, Integrating Derivatives | **6.1-6.5** |
| 38-40 | To define L^p spaces and study its completeness property. | Youngs inequality, Holder inequality, Minikowski inequality, and Reisz-Fischer theorem | **7.1-7.4** |
| Approximations and Separability – Self study |

**5. Evaluation Scheme:**

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| --- | --- | --- | --- | --- | --- |
| **EC No.** | **Evaluation Component** | **Duration** | **Weightage\*** | **Date and Time** | **Nature of Component** |
| 1. | Quiz 1 | To be announced | 5% | To be announced | Closed Book |
| 2. | Assignment 1 | To be announced | 10% | To be announced | Open Book |
| 3. | Mid-Semester Test | 90 min | 30% | 12/03 - 11.00 - 12.30PM | Closed Book |
| 4. | Quiz 2 | To be announced | 5% | To be announced | Closed Book |
| 5. | Assignment 2 | To be announced | 10% | To be announced | Open Book |
| 6. | Comprehensive Examination | 3 hours | 40% | 09/05 FN | Closed Book |

**6. Chamber Consultation Hour:**To be announced in the class.

**7. Notices:**All notices concerning this course will be displayed in CMS or through e-mail only.

**8. Make-up Policy:**

* Makeup will be given only for very genuine cases and prior permission has to be obtained from the Instructor-in-charge.

**9. Academic Honesty and Integrity Policy:** Academic honesty and integrity are to be maintained by all the students throughout the semester and no type of academic dishonesty is acceptable.

**INSTRUCTOR-IN-CHARGE**